A NEW CONVERTER TRANSFORMER DESIGN TECHNIQUE

RELATES

COMPLEX PERFORMANCE PARAMETERS

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Abstract

Volt amperes available at the output terminals of an AC power transformer can be expressed as a function of percent regulation and six other general transformer variables. The effects of leakage reactance and waveshape of the applied voltage are discussed.

1. INTRODUCTION

Conventional transformer design (manual and computerized) proceeds with the designer or computer selecting a core size based primarily on volt amperes, frequency, and temperature rise. Furthermore, core size is generally determined by certain empirical methods such as volt amperes per pound of core at a specific frequency or relative power handling capacity expressed as a product of core cross sectional area and window area. The percentage regulation inherent to most transformer designs is not known until the design is nearly complete, and once known, may often dictate redesign on either a larger or smaller core. Percent regulation effects the size of any transformer considerably, and the analysis of the interrelationship between percent regulation, size, and available volt amperes has been the principal reason for this research.

Secondarily, the analysis quantifies all transformer parameters supporting available volt amperes except temperature rise... the analysis of which involves a different system of curves and will not be treated analytically in this discussion.

2. PRELIMINARY CONDITIONS

Before beginning the model analysis, the following conditions are assumed to be in effect:

(1) An infinite amount of volt amperes is available to the transformer primary terminals... the transformer is under evaluation... not the source.

(2) The transformer is a simple one-to-one turns ratio device with N turns on the primary and N turns on the secondary... it can be demonstrated the analysis works for transformers with any turns ratio.

(3) Primary resistance is R ohms and secondary resistance is R ohms... although this is not exactly practical, the variance is insignificant to this analysis.

(4) Total leakage reactance is $2X_L$... as would be measured in short circuit inductance testing.

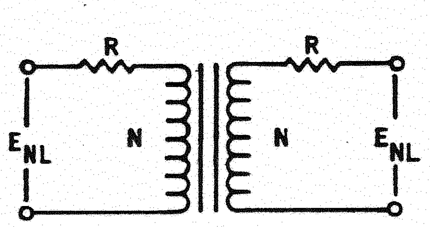
(5) Core loss is small and open circuit inductance is high... this is a very common transformer requirement.

(6) Temperature effects will not be discussed for the sake of brevity.

(7) The development to follow is accomplished using a square shaped voltage waveform... any waveshape can be evaluated providing the relationship between RMS voltage and no load flux density is known.

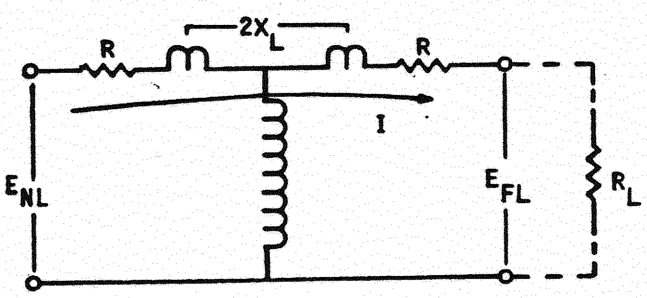
3. TRANSFORMER ANALYSIS

Given the transformer of Figure 1 and its equivalent circuit of Figure 2, the following relationships may be developed.



A SIMPLE 1/1 TURNS RATIO TRANSFORMER (UNLOADED)

FIGURE 1



EQUIVALENT CIRCUIT OF FIGURE 1 (LOADED WITH R_L OHMS AND ASSUMING NEGLIGIBLE CORE LOSS)

FIGURE 2

If percent regulation is defined as:

$$\%REG = \left[\frac{E_{NL}-E_{FL}}{E_{FL}}\right]100 \qquad \text{or,} \qquad (1)$$

$$\%REG = \frac{\left[2IR+j2IX_L\right]100}{E_{NL}-2IR-j2IX_L} \qquad (2)$$

Then expressions for current, resistive and reactive voltage drops in terms of no load voltage and %regulation are are follows:

The current expression...

$$I = \frac{\%REG\ E_{NL}}{\left[2R+j2X_L\right]\left[\%REG+100\right]} \qquad (3)$$

The expression for resistive voltage drop...

$$IR = \frac{\%REG\ E_{NL}}{\left[2+j\frac{2X_L}{R}\right]\left[\%REG+100\right]} \qquad (4)$$

The expression for reactive voltage drop...

$$IX_L = \frac{\%REG\ E_{NL}}{\left[\frac{2R}{X_L}+j2\right]\left[\%REG+100\right]} \qquad (5)$$

An expression for volt amperes available to the load resistance, R_L, in terms of no load voltage, current, resistive and reactive voltage drops is:

$$VA = \left|E_{NL}-2IR-j2IX_L\right|I \qquad (6)$$

Substituting the current and voltage drop terms into equation (6)...

$$VA = \left[E_{NL} - \frac{2\%REG\ E_{NL}}{\left[2+j\frac{2X_L}{R}\right]\left[\%REG+100\right]} - \frac{j2\%REG\ E_{NL}}{\left[\frac{2R}{X_L}+j2\right]\left[\%REG+100\right]}\right]\frac{\%REG\ E_{NL}}{\left[2R+j2X_L\right]\left[\%REG+100\right]} \qquad (7)$$

Placing the terms inside the brackets over a least common denominator and factoring the common terms,

$$VA = \left[\frac{\left[\frac{R}{X_L}+j\right]\left[1+j\frac{X_L}{R}\right]\left[\%REG+100\right] - \%REG\left[\frac{R}{X_L}+j\right] - j\%REG\left[1+j\frac{X_L}{R}\right]}{\left[\frac{R}{X_L}+j\right]\left[1+j\frac{X_L}{R}\right]}\right]\left(\frac{1}{2}\right)\frac{\%REG\ E_{NL}^{\ 2}}{\left[R+jX_L\right]\left[\%REG+100\right]^2} \qquad (8)$$

Which reduces to:

$$VA = \frac{\left[100\frac{R}{X_L}+j200+j^2 100\frac{X_L}{R}\right]}{\left[\frac{R}{X_L}+\ j2+j^2\frac{X_L}{R}\right]}\left(\frac{1}{2}\right)\frac{\%REG\ E_{NL}^{\ 2}}{\left[R+jX_L\right]\left[\%REG+100\right]^2} \qquad (9)$$

or

$$VA = \frac{50\ \%REG\ E_{NL}^{2}}{\left[R+jX_L\right]\left[\%REG+100\right]^{2}} \qquad (10)$$

Defining an expression for no load voltage from the Faraday equation for square or rectangular voltage waveshape:

$$E_{NL} = 3.99\left[\frac{1}{.3937}\right]^{2} B_{NL}\ f\ A_c\ K\ N\ 10^{-8} \qquad (11)$$

Substituting equation (11) into equation (10) and normalizing the demoninators ...

$$VA = \frac{50\times3.99^{2}\times10^{-16}}{10000}\left[\frac{1}{.3937}\right]^{4} \frac{B_{NL}^{2} f^{2} A_c^{2} K^{2}\left(N^{2}/R\right)\%REG}{\left[1+j\frac{X_L}{R}\right]\left[1+\frac{\%REG}{100}\right]^{2}} \qquad (12)$$

It is now convenient to define turns and resistance in terms of core and wire parameters:

$$N = \frac{A_p}{d^2} = \left[\frac{A_p}{A_w}\right]\frac{A_w}{d^2} \qquad (13)$$

and

$$R = MTL\left[\frac{A_p}{A_w}\right]\frac{A_w}{d^2}\ \rho/12000 \qquad (14)$$

Substituting equations (13) and (14) into equation (12) ...

$$VA = \frac{12000\times50\times3.99^{2}\times10^{-16}}{10000}\left[\frac{1}{.3937}\right]^{4}\left[\frac{A_p}{A_w}\right]\frac{B_{NL}^{2} f^{2} K^{2} A_c^{2} A_w\ \%REG}{\rho d^{2}\ MTL\left[1+j\frac{X_L}{R}\right]\left[1+\frac{\%REG}{100}\right]^{2}} \qquad (15)$$

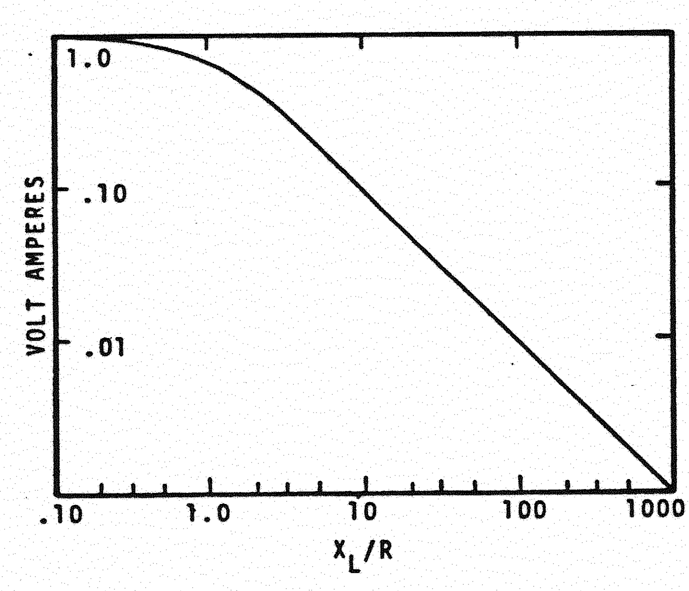
Flux density as used in equation (11) was in dimensional units of Gauss. Converting to Kilogauss to increase the size of the numerical coefficient yields the general form of the VA equation ...

$$VA = 3.99\times10^{-6}\left[\frac{A_p}{A_w}\right]\frac{B_{NL}^{2} f^{2} K^{2} A_c^{2} A_w\ \%REG}{\left[\frac{\rho d_p^{2}+\rho d_s^{2}}{2}\right] MTL\left[1+\left(\frac{X_L}{R}\right)^{2}\right]^{\frac{1}{2}}\left[1+\frac{\%REG}{100}\right]^{2}} \qquad (16)$$

It is convenient to plot a rolloff curve for the subproduct $\frac{1}{\left[1+\left(\frac{X_L}{R}\right)^2\right]^{1/2}}$ as shown in Figure 3. Equation (16) may then be simplified to:

$$VA = 3.99\times10^{-6}\left[\frac{A_p}{A_w}\right]\frac{B_{NL}^{2} f^{2} K^{2} A_c^{2} A_w \%REG}{\left[\frac{\rho d_p^{2}+\rho d_s^{2}}{2}\right] MTL \left[1+\frac{\%REG}{100}\right]^{2}} \qquad (17)$$

For low frequencies X_L/R is insignificant and $\frac{1}{\left[1+\left(\frac{X_L}{R}\right)^2\right]^{1/2}}$ is very nearly one. For high frequencies it is convenient to work with equation (17) and apply the rolloff curve of Figure 3 to the final result.



VOLT AMPERES
VERSUS
RATIO OF LEAKAGE REACTANCE
TO
WINDING RESISTANCE

FIGURE 3

4A. DEFINITION OF TERMS

A_c	Core cross sectional area in inches squared
$\left[\frac{A_p}{A_w}\right]$	Ratio of the area of primary winding to the total window area. A pure number
A_w	Total window area in inches squared
B_{NL}	No load flux density, in Gauss throughout derivation, changed to Kilogauss at equation 16 to increase size of the numerical coefficient
d	Diameter of magnet wire in inches
f	Frequency in hertz
E_{FL}	Full load output voltage in volts RMS
E_{NL}	No load output voltage in volts RMS
I	Full load current in amperes RMS
j	Defines an imaginary number system in quadrature with the axis of percent regulation in j1, j2, j3... jn units
K	Stacking factor of core. A pure number
MTL	Mean length of a turn of wire in inches. MTL as used in equation is actually the average of primary and secondary windings mean lengths of turn
N	Number of primary turns
ρd^2	Resistivity of wire in ohms per 1000 feet (or 12000 inches) times outside diameter of wire (over in-

4A. DEFINITION OF TERMS (CONT'D)

sulation) in inches squared

R — Resistance of primary winding in ohms

R_L — Resistance of a load on transformer output terminals in ohms

%REG — %Regulation= $\left[\frac{E_{NL}-E_{FL}}{E_{FL}}\right]100$ a percentage or pure number

VA — Available or transferable volt amperes at the output terminals of the transformer in volts RMS times amperes RMS

$2X_L$ — Transformer leakage reactance in ohms

4B. DIMENSIONAL ANALYSIS OF THE VOLT AMPERE EQUATION

Utilizing basic dimensional units of length l, mass m, time t, and permeability μ:

$$VA \sim \left[\frac{A_p}{A_w}\right]\left[B^2\right]\left[f^2\right]\left[\frac{K^2 A_c^2 A_w}{MTL}\right]\left[\frac{1}{\rho d^2}\right]$$

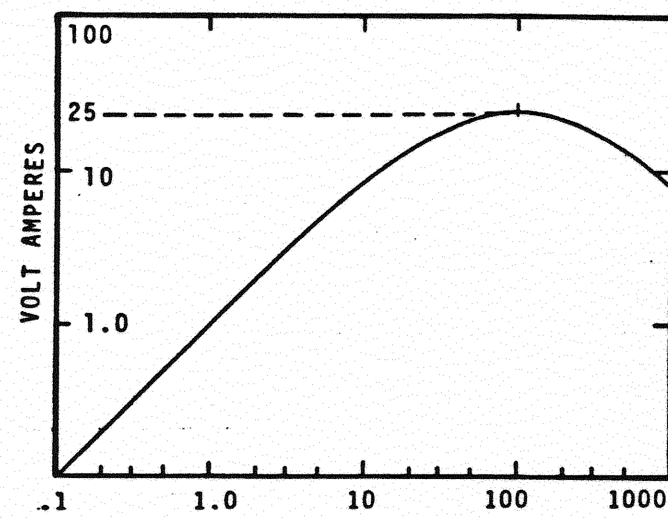
$$VA = \left[\frac{l^2}{l^2}\right]\left[\frac{m\mu}{l t^2}\right]\left[\frac{1}{t^2}\right]\left[\frac{l^6}{l}\right]\left[\frac{t}{\mu l^2}\right]$$

$$VA = \frac{ml^2}{t^3}$$ ... which is consistant with power units.(1)

5. VOLT AMPERES VERSUS PERCENT REGULATION

The shape of volt amperes versus percent regulation is explained by $VA \sim \frac{\%REG}{\left[1+\frac{\%REG}{100}\right]^2}$.

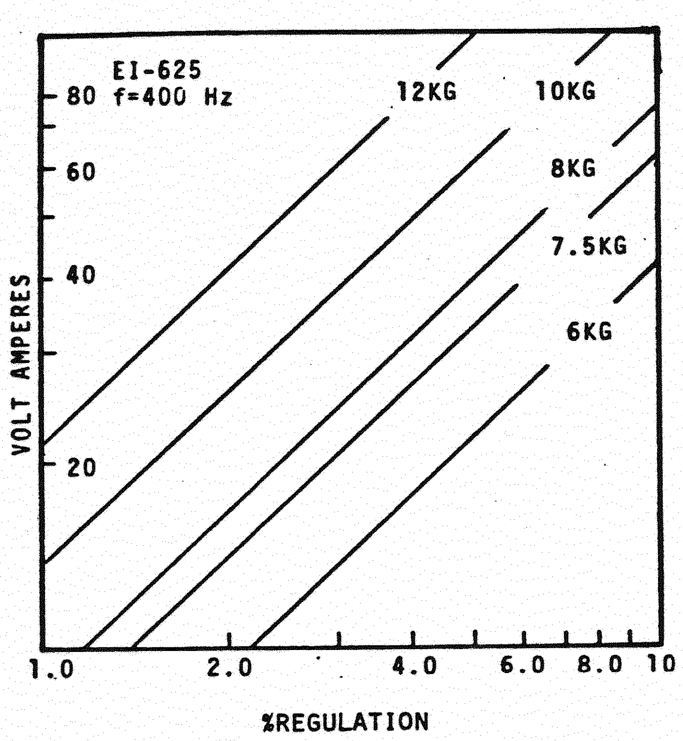
A curve of this function versus percent regulation is as shown in Figure 4. The function maximizes at 100 percent regulation and falls off asymptotically with increasing percent regulation. Operation at percent regulations above approximately 25% is limited to fault or short circuit conditions, accompanied by rapid temperature rise and low efficiency. It is possible however, to analyze volt amperes available under these conditions.



VOLT AMPERES VERSUS %REGULATION

FIGURE 4

Equation (17) may be used to test volt amperes versus percent regulation for a given size under a wide variety of transformer conditions. Families of flux density curves may be plotted for a specific size at a specific frequency as shown in Figure 5. It is also possible to attach various general ranges of percent regulation for different frequencies. This is not meant to imply designs cannot be outside these limits, only, that typical values of percent regulation are less as frequency increases due to reduced number of turns and winding resistance. Table 1 lists typical ranges of %regulation versus



VOLT AMPERES VERSUS %REGULATION
FOR VARIOUS FLUX DENSITIES

FIGURE 5

frequency. Above this range of %regulation the copper loss is high and out of balance with the core loss. Below this range the core loss may be high and out of balance with the copper loss and/or the transformer becomes larger than necessary.

TABLE 1

FREQUENCY (HERTZ)	%REGULATION
60	5 to 20
400	1 to 10
2400	.5 to 5
15000	.2 to 1
30000	.1 to 1
100000	.1 to .8

6. SIGNIFICANCE OF THE REMAINING SUBPRODUCTS

While the reactance to resistance ratio and percent regulation functions determine the shape of available volt amperes, the remaining subproducts each contribute to the magnitude. In other words, volt amperes are proportional to each of these factors.

$VA \sim 3.99 \times 10^{-6}$ ... The Numerical Coefficient

$VA \sim \left[\frac{A_p}{A_w}\right]$ ... The Window Utilization Factor

$VA \sim B_{NL}{}^2 f^2$ ... The Magnetic Utilization Factor

$VA \sim \left[\frac{1}{\frac{\rho d_p{}^2 \; \rho d_s{}^2}{2}}\right]$ ... The Wire Insulation Figure of Merit (Average of Primary and Secondary Wire Size)

$VA \sim \frac{K^2 A_c{}^2 A_w}{MTL}$ ... The Core Geometry Factor

6.1 NUMERICAL COEFFICIENT

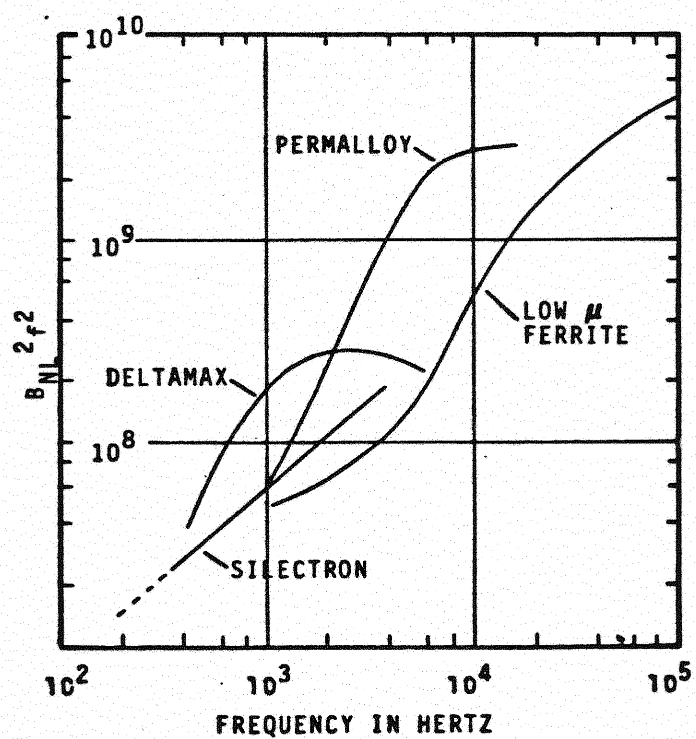
This coefficient is a natural product of the Faraday constant and other constants which factored out of the VA equation; see equation (15). When the voltage waveform is sinusoidal it can be demonstrated the numerical coefficient becomes 4.9232905×10^{-6}. As the relationship between RMS voltage and flux density changes, this constant also changes.

6.2 WINDOW UTILIZATION FACTOR

The ratio of area occupied by the primary winding (which carries all the volt amperes) to the total window area. Toroidal transformers have typical ratios between .10 to .20 while scrapless laminations have ratios between .20 to .30, and three phase "E" cores have ratios between .075 to .15.

6.3 MAGNETIC UTILIZATION FACTOR

The magnetic utilization factor $B_{NL}^2f^2$ explains the reason a magnetically cored device transfers power. Without being able to reach high magnetic flux densities in the various types of ferromagnetic core materials, at the frequencies they are energized, the transfer of power by magnetic transformer action could not occur. At low frequencies, one ferromagnetic material may be operated at a higher flux density than another before saturation occurs. The material which can operate at the higher flux density can then transfer more volt amperes. At high frequencies, one ferromagnetic material may be operated at a higher flux density than another before the core loss becomes excessive. Again the material which can operate at the higher flux density under these conditions can transfer more volt amperes. Figure 6 is a plot of $B_{NL}^2f^2$ versus f for some of the more commonly used ferromagnetic materials.



THE MAGNETIC UTILIZATION FACTOR $B_{NL}^2f^2$ VERSUS FREQUENCY FOR VARIOUS CORE MATERIALS

FIGURE 6

6.4 WIRE INSULATION FIGURE OF MERIT

This factor is a measure of the spatial efficiency of the wire used. For bare wire it is constant. For various types of film covered magnet wires ρd^2 slowly increases as wire size decreases, since the film insulation occupies an increasing percentage of the total wire area. Table 2 illustrates typical values of ρd^2. The average of all wire sizes ρd^2 should be used in the VA equation. This factor is also temperature sensitive, varying as the resistivity of the wire alloy, with temperature.

AWG #	ρd^2 BARE COPPER	ρd^2 SINGLE FILM	ρd^2 HEAVY FILM
10	.01037	.01088	.01124
15	.01038	.01119	.01172
20	.01037	.01156	.01238
25	.01038	.01205	.01345
30	.01040	.01279	.01468
35	.01036	.01306	.01563
40	.01034	.01382	.01616

WIRE INSULATION FIGURE OF MERIT

TABLE 2

6.5 CORE GEOMETRY

Core geometry contributes the factor $\frac{K^2 A_c^{\,2} A_w}{MTL}$ to the available volt amperes product. It is significant that both power and this factor have units of length to the fifth power. This term also demonstrates core area, A_c, effects available volt amperes an exponential order higher than window area, A_w. Cores of different shapes, sizes and stacking factors may be compared relative to available volt amperes, and relative to one another by this relationship.

7. CONCLUSION

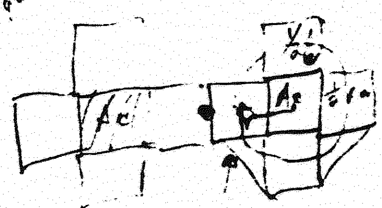
The mathematical interrelationship between available volt amperes, percent regulation, and transformer size has been explored, and inherent transformer percent regulation may be predicted prior to design. Furthermore, an expression has been developed which is general to AC power transformers: it is flexible enough to analyze such widely diverse types of devices as high frequency square wave converter transformers, low frequency sinusoidally excited, single or multi-phase transformers, audio, auto and wide band AC transformers. It provides an explanation of how available volt amperes are effected by leakage reactance. It is specific enough to describe the effect of an incompletely filled window, the use of different wire film insulation thicknesses, or different wire alloys on available volt amperes. It demonstrates the merits of core and wire size and the magnetic properties of the core alloy. Volt amperes available under fault or short circuit conditions may be analyzed and many relative situations may be modeled from the different subproducts.

The adaptability of this volt ampere equation should provide a valuable tool to the entire magnetics industry.

8. REFERENCES

1. Warren B. Boast, "Principles of Electric and Magnetic Fields", Second Edition, March 1956, p. 412

Area circle = πr^2
Area enclosed ≈ $A_c + K1$
$MTL = 2\pi\sqrt{\frac{1}{\pi}(A_c + 3A_w K1)} + 2x\pi...Ht$ non square



← toroid

A_c +