

Inches to the Fifth

The Electrical Way To Design Magnetics

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$$\frac{(LI)^2}{DCR} = Kc * (Ur H)^2 * \frac{Ac^2 Aw}{MTL} * \frac{1}{pd^2} * \frac{Ap}{Aw}$$

The Inches to the Fifth Equation

$$(LI)^2 / DCR = Kc * (Ur H)^2 * (Ac^2 * Aw) / MTL * 1 / pd^2 * Ap / Aw$$

Why is this equation your friend?

This equation was named “Inches to the Fifth” by Lowell Quist because the dimensional units for the core (inches) is raised to the fifth power.

Agenda

$$\frac{(LI)^2}{DCR} = K_c * (U_r H)^2 * \frac{A_c^2 A_w}{MTL} * \frac{1}{pd^2} * \frac{A_p}{A_w}$$

- Review Magnetics Fundamentals
- Short Discussion of Tolerances
- Inch⁵ Design Equation
- Design Example 1
- Design Example 2
- Bonus Slides

Power in = Power out

$$\frac{(LI)^2}{DCR} = Kc * (Ur H)^2 * \frac{Ac^2 Aw}{MTL} * \frac{1}{pd^2} * \frac{Ap}{Aw}$$

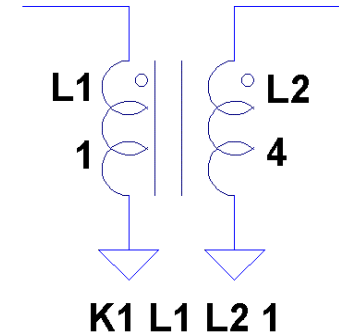
For a transformer, Power in = Power out (ignoring losses)

Or

The Ampere Turns into the "DOT" sums to zero.

$$N1 * I1 = N2 * I2 + N3 * I3 \dots$$

$$1 \text{ turn} * 4 \text{ amps in} = 2 \text{ turns} * 2 \text{ amps out}$$



L1 is 1 turn
L2 is 2 turns

Inductance is proportional to turns squared.

So in a SPICE model, 2 turns has 4 times the inductance as 1 turn in the part model.

Advanced, in a given cycle :

Energy in = Energy out – Energy lost – Change in stored energy.

Energy in = Energy out in a good design.

Dot Convention

$$\frac{(LI)^2}{DCR} = K_c * (U_r H)^2 * \frac{A_c^2 A_w}{MTL} * \frac{1}{pd^2} * \frac{A_p}{A_w}$$

Looking at the end of the winding (cross section of the core):

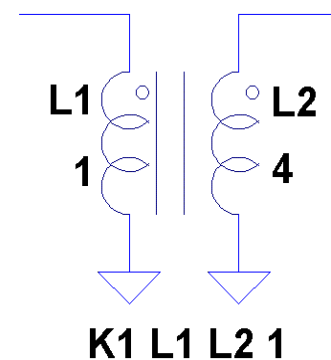
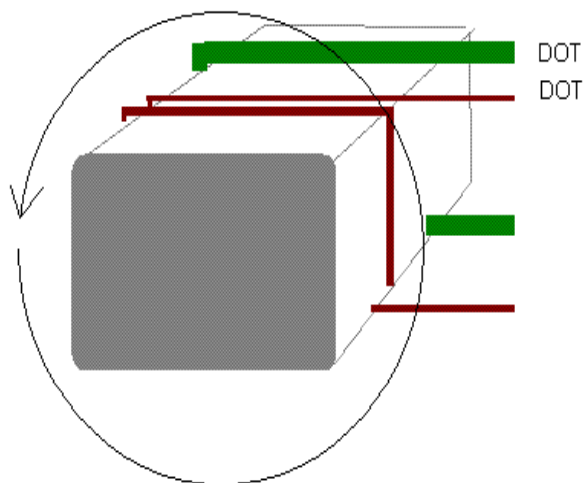
- All wire ends that go the same direction around the cross section of the core have the same voltage polarity.
- All windings on a given leg of a magnetic have the same volts/turn independent of the direction of current flow in or out of the dot.

So pick one lead and call it the “dot”. All leads that go the same direction around the center of the core are “dotted” ends of the winding.

With 1V peak across L1, L2 will have 2V peak across it.

Remember, the SPICE model lists inductance per winding, not the turns. Inductance follows turns squared.

Note: Some manufactures will pick the “dot” to also indicate the lead that is closest to the core or closest to another winding. There isn't a standard for this. Just be aware it exists.



L1 is 1 turn
L2 is 2 turns

Average Voltage = 0

$$\frac{(LI)^2}{DCR} = Kc * (Ur H)^2 * \frac{Ac^2 Aw}{MTL} * \frac{1}{pd^2} * \frac{Ap}{Aw}$$

$E = L di/dt + I dL/dt$ (the 2nd Term is usually dropped.) If E is not zero, the current rises forever (or at least until the resistance of the circuit drops the voltage to zero.)

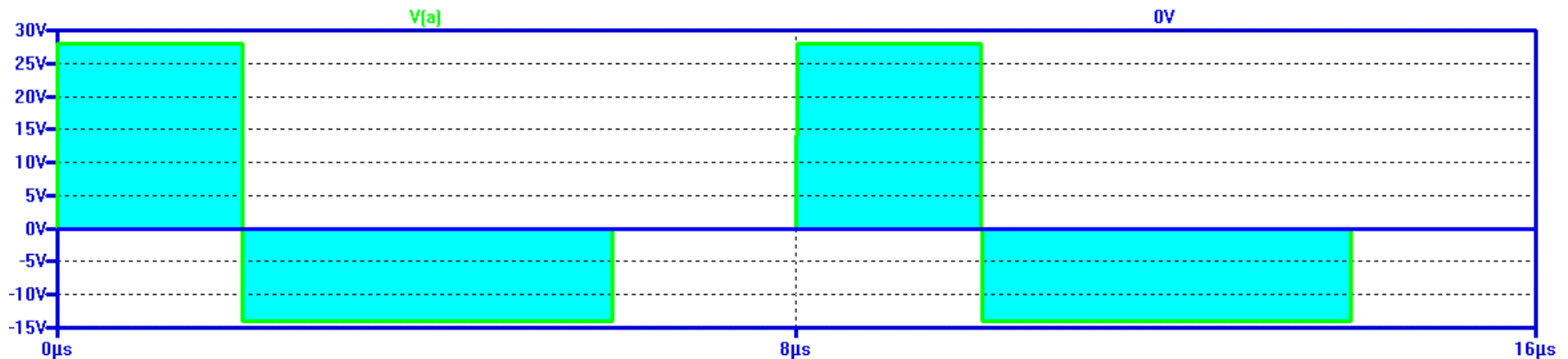
Example of a single winding in steady state

$$E1 * T1 + E2 * T2 + E3 * T3 = 0 \text{ where } T1 + T2 + T3 = \text{one period}$$

Or

$$E1 * T1 = -E2 * T2 - E3 * T3$$

$$28V * 2 \text{ usec} = -(-14V) * 4 \text{ usec} + 0V * 2 \text{ usec}.$$



The Continuous Conduction transformer equation (no time spent at zero volts) :

$$E1 \frac{T1}{N1} = -E2 \frac{T2}{N2}$$

Gauss (cgs)

$$\frac{(LI)^2}{DCR} = Kc * (Ur H)^2 * \frac{Ac^2 Aw}{MTL} * \frac{1}{pd^2} * \frac{Ap}{Aw}$$

Flux Level in Gauss (Bmax is the zero to peak value)

$$B_{max} = E_{rms} * 1E8 / (4.44 * F * N * A_c) \quad \text{Sine wave}$$

$$B_{max} = E_{pk} * 1E8 / (4.00 * F * N * A_c) \quad \text{Square Wave}$$

Inductance

$$L = 0.4 * PI * 10E-8 * U_r * N^2 * A_e / l_m$$

$$L = AL * (N/1000)^2 \text{ mH} \quad (AL \text{ in mH per } 1000 \text{ turns})$$

$$L = AL * N^2 \text{ nH} \quad (AL \text{ in nH per turn})$$

Oersted (Field strength)

$$H = 0.4 * PI * N * I / l_m$$

Units for the above equations:

L Inductance in henries

l_m Magnetic path length in centimeters

A_c Actual iron cross-sectional area in square centimeters

H Oersteds

I Amps

N Turns

I Current in Amps

F Frequency in Hertz

B Gauss

SRF (Self Resonant Frequency)

$$\frac{(LI)^2}{DCR} = Kc * (Ur H)^2 * \frac{Ac^2 Aw}{MTL} * \frac{1}{pd^2} * \frac{Ap}{Aw}$$

We would think that something as simple as SRF would not cause problems.

$$f = 1 / [2 * pi * Sqrt (L_{inductor} * C_{parasitic})]$$

It does.

This is because L (inductance) changes with frequency. The better the material is, the less it changes. An improvement in the core material can cause SRF failures. With the same parasitic capacitance in the part, if we switch from 160u High Flux to 160u MPP, the SRF will drop. This effect has caused incoming inspection failures on parts that actually worked better than before.

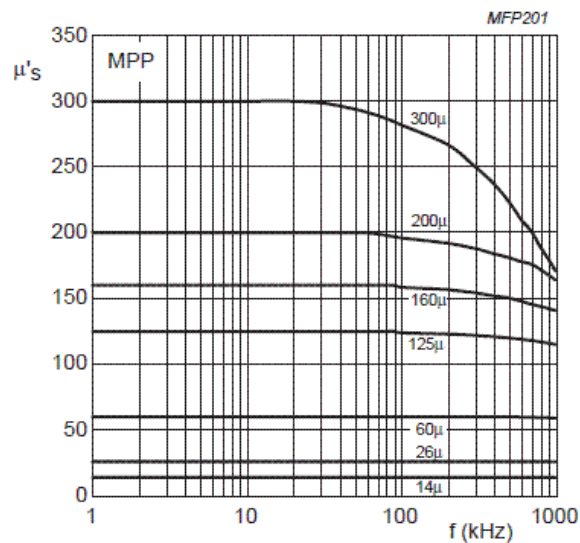


Fig.1 Initial permeability as a function of frequency.

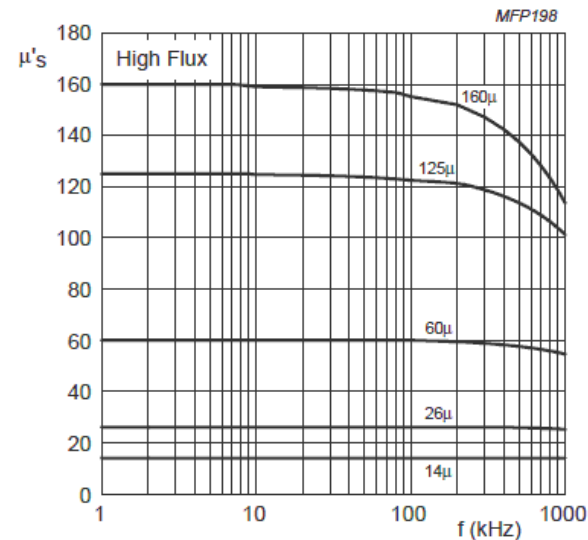


Fig.1 Initial permeability as a function of frequency.

High flux 160u is ~115u at 1 MHz, MPP 160u is ~145u. Both are dropping in perm above 1 MHz. For the same parasitic capacitance, the SRF for High flux will be higher than the MPP because it's inductance has dropped more. This gives the false impression that the High Flux part has less parasitic capacitance.

Curves from Magnetics Inc website.

Tolerances 1

$$\frac{(LI)^2}{DCR} = K_c * (U_r H)^2 * \frac{A_c^2 A_w}{MPL} * \frac{1}{pd^2} * \frac{A_p}{A_w}$$

Typical Room Temperature Tolerances of RAW core materials

Raw Inductance before measurement errors

MPP	+/- 8%	(at 0 DC bias), (tighter cores available for \$\$)
Ferrite	+/- 20%	(low u)
Ferrite	+/- 30%	(high u) (drive dependent)
Tape Wound	+100% / - 50%	(extremely drive dependent)

Other

Wound DC Resistance	+/- 14%	
Saturation Voltage	+15 /- 5%	
Turns	+/- 1%	(1 turn out of 100 on toroids)

Expect a 3 - 6% Instrumentation error measuring saturation

Expect a 0.5 - 10% Instrumentation error measuring inductance

Expect problems measuring inductance with $X_L (2 * \pi * F * L) < \sim 1 \text{ ohm}$

Remember to put the LCR meter in the “parallel” or “series” mode that matches the dominant loss mechanism (series loss = DCR, parallel loss = core loss.)

Tolerances 2

$$\frac{(LI)^2}{DCR} = K_c * (U_r H)^2 * \frac{A_c^2 A_w}{MTL} * \frac{1}{pd^2} * \frac{A_p}{A_w}$$

Making the specifications more stringent than necessary you will:

1. Increase cost
2. Lower Yield
3. Increase delivery time

A few examples are:

1. Specifying a lead length to be 4.0" +/- 0.1 when 4.5" +/- 0.6 would work.
2. Specifying inductance to be 1.0 +/- 0.2 Henries when 0.8 Henries minimum would work.
3. Specifying a window on DCR, SRF, leakage, inductance with DC, Capacitances etc. instead of specifying a one side limit only. Turns ratio and inductance at zero DC are usually all that needs to be specified with a window.

Life limiters in Magnetics

$$\frac{(LI)^2}{DCR} = K_c * (U_r H)^2 * \frac{A_c^2 A_w}{MTL} * \frac{1}{pd^2} * \frac{A_p}{A_w}$$

- Voltage Stress

- Punch through
- Corona
- Increased temperature rise

- Temperature

- Degradation of the insulation
- Changes in losses

- Mechanical Stress

- Fatigue breakage
- Abrupt failure

Note:

1. **Current is not listed** as a “life killer.” High current for a short period of time is just a transient thermal problem to solve. A circular mil/ amp limit is a design aid, not a design limit.
2. Saturation from current is circuit behavior. Saturation in a core can be a good thing. It can lower the output overshoot in an LC tank from a step input voltage at the expense of increased inrush current.

One known exception to “we don’t care about peak current” is cobalt doped powered iron used in RF coils. High magnetic fields can pull the cobalt out of the crystal structure.

Design Methods History

$$\frac{(LI)^2}{DCR} = K_c * (U_r H)^2 * \frac{A_c^2 A_w}{MTL} * \frac{1}{pd^2} * \frac{A_p}{A_w}$$

- Typically transformers are designed with application curves using Inch⁴ and inductors are designed with Hanna Curves. The problems with these methods are:
 - They use correction factors for free convection. This isn't much use for other cooling methods or lower or higher temperature rises
 - Does not allow for correction for wiring resistance.
 - Difficult to use for high crest factor designs
 - ♦ Neither have regulation (or dc resistance) as a free variable.

A better solution: Inch⁵

Lowell Quist originally presented inches to the fifth (in⁵) as a regulation based transformer design process.

Paul Schwerman extended the equations to inductors and presented it at Sixteenth International PCI Conference, October 1988

$$\frac{(LI)^2}{DCR} = K_c * (U_r H)^2 * \frac{A_c^2 A_w}{MTL} * \frac{1}{pd^2} * \frac{A_p}{A_w}$$

- **The Inches to the Fifth Equation**

$$(LI)^2 / DCR = K_c * (U_r H)^2 * (A_c^2 * A_w) / MTL * 1 / pd^2 * A_p / A_w$$

- **Assumptions used in the equation:**

- **The core is reasonably linear.**

On a linear structure,

1. $L * I = E * T$ (from $E = L di/dt$)
2. U_r is constant
3. $U_r (\text{perm}) * H (\text{oersted}) = \text{gauss}$.
4. Small deviations from linearity won't break the design.

- **Permeability is a free variable. . . We can pick any perm core we want.**

This equation was derived in the paper "Single Iteration Magnetic Design Based On Winding Resistance" Sixteenth International PCI Conference, October 1988, pages 222-234, Paul Schwerman.

Inductor Considerations

$$\frac{(LI)^2}{DCR} = K_c * (U_r H)^2 * \frac{A_c^2 A_w}{MTL} * \frac{1}{pd^2} * \frac{A_p}{A_w}$$

- | | | |
|--|----------|-----------------|
| 1. Inductance | L | Electrical |
| 2. Current Response | L(gauss) | Electrical |
| 3. DC resistance | DCR | Electrical |
| 4. AC losses | Q | Electrical |
| 5. Self Resonant Frequency | SRF | (be careful!) |
| 6. Final Dimensions | | |
| 7. Removal of heat loss
(temperature rise and cooling method) | | |
| 8. Corona and insulation breakdown | | |

The first three design constraints above directly feed into the “inches to the fifth” design process.

The last five parameters are indirectly controlled by the Inch⁵ equation through DCR, UrH (gauss), and Ap/Aw

Insights - $(LI)^2 / DCR$ Factor

$$\frac{(LI)^2}{DCR} = K_c * (U_r H)^2 * \frac{A_c^2 A_w}{MTL} * \frac{1}{pd^2} * \frac{A_p}{A_w}$$

- On the same base material and same size part:
 - At a given dc current, to half the DCR, the inductance has to drop by a factor of 0.707.
 - At a given dc current, to double the inductance, the DCR has to go up a factor of 4.
- $L * I$ is the same as $E * T$. The area under a voltage pulse can be directly substituted for $L * I$ to use this for transformer design.
- The term K_c is the design constant that accumulates all the dimensional unit multipliers (cm to inches) and design coefficients like U_o , ohm/foot etc.

Insights – Ur H (gauss) Component $\frac{(LI)^2}{DCR} = Kc * (Ur H)^2 * \frac{Ac^2 Aw}{MTL} * \frac{1}{pd^2} * \frac{Ap}{Aw}$

- **On the same size part:**

- **At a given DCR and inductance:**

- ◆ Using a material that supports 24kg vs.. 16kg allows 50% more current before saturation.
- ◆ 24 kg/ 15 kg= 1.5
- ◆ 1 tesla = 10,000 gauss

- **At a given dc current and inductance:**

- ◆ Using a material that supports 24kg vs.. 16kg allows 1 / 2.25 the DCR.
- ◆ $1.5^2 = 2.25$

- **At a given DCR and current:**

- ◆ Using a material that supports 24kg vs.. 16kg allows 50% more inductance.

Insights – Ac²Aw/MTL (Core Shape)

$$\frac{(LI)^2}{DCR} = K_c * (U_r H)^2 * \frac{Ac^2 Aw}{MTL} * \frac{1}{pd^2} * \frac{Ap}{Aw}$$

- **Ac is area of the core. This term is squared. For a given size part, it is desirable to trade core area for window area.**
- **Aw is the window area, the area where we can put windings .**
- **MTL is the mean turn length or average length of one turn of wire.**
Hint: When possible, put the windings with higher currents on the core first so they have a turn length shorter than the average “MTL.”
- **As the area of the core increases, so will the MTL. A round core area is optimum because it has the lowest perimeter (MTL) for a given core area. But a round center leg on a core is not always the best choice or even available off the shelf.**
- **On C-Cores, a dual bobbin arrangement usually gives better performance through a lower MTL and a lower leakage inductance. The lower leakage inductance is from having a longer effective bobbin width because using two bobbins.**
 - **Knowing “There Ain’t No Such Thing As A Free Variable,”
what is the price for using a dual bobbin C-Core? . . . Increased capacitance. . .**

Insights – 1/pd² (Wire Factor)

$$\frac{(LI)^2}{DCR} = Kc * (Ur H)^2 * \frac{Ac^2 Aw}{MTL} * \frac{1}{pd^2} * \frac{Ap}{Aw}$$

Usually, I just use 0.013 for pd² because pd² doesn't change a lot with current for normal magnet wire.

pd² for various wire types (p in mOhm/ft, d in inches)

AWG #	pd ² Round Solid Bare Copper	pd ² Round Solid Double Film	pd ² Square Solid Double Film	pd ² Round Solid 81822 Teflon	pd ² Round Stranded 22759 Teflon	pd ² Round Aluminum Double Film
4	.01037	.01094	.00901			.01770
12	.01037	.01139	.00991		.02352	.01838
16	.01037	.01193			.02680	
22	.01037	.01279		.04225	.03928	
24	.01037	.01323		.05188	.04921	
28	.01037	.01411		.05410	.07816	
36	.01037	.01646				
40	.01037	.01726				

Note: pd² for copper foil without insulation = .00814

From "Single Iteration Magnetic Design Based On Winding Resistance"

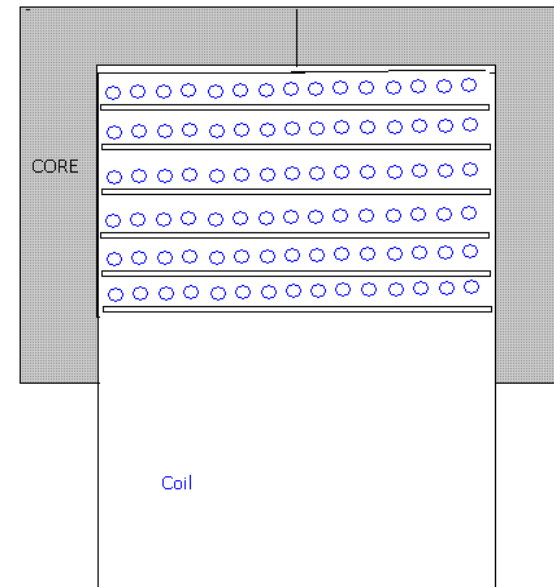
Insights – Ap/Aw Fill Factor

$$\frac{(LI)^2}{DCR} = Kc * (Ur H)^2 * \frac{Ac^2 Aw}{MTL} * \frac{1}{pd^2} * \frac{Ap}{Aw}$$

- **Ap/Aw (fill factor) allows for coupled inductors, margin for bobbins, high voltage insulation and for allocating production mechanical margin.**
- Ap is the area of the primary winding. Aw is the area of the minimum available winding window. On a simple inductor where there is only one winding, that winding occupies Ap. On a coupled inductor (or flyback), the winding you are picking the inductance and DCR on is the Ap winding. (See example 2)
- Do not “force” the fill factor to be a specific number. Ap/Aw is a figure of merit used in the beginning of the design. Having all the wire and insulation fit on the worst case smallest window area is the true requirement.

A window that has 0.5 square inches of available area (Aw) with 0.4 square inches of total copper in the window (Ap) has a fill factor of 80%. If after worst case wire bulge and insulation etc. is added in, the window had 10 mils of margin to each edge of the window, the design would be producible.

If this were a transformer with ½ the copper primary and ½ the copper secondary, AP/AW would be 40% (80% * ½) and the DCR used in the equation would be the DCR for the primary.



Insights – Ap/Aw Fill Factor

$$\frac{(LI)^2}{DCR} = K_c * (U_r H)^2 * \frac{A_c^2 A_w}{MTL} * \frac{1}{pd^2} * \frac{A_p}{A_w}$$

- **On bobbin wound** (not on a toroid):
 - **The winding width occupied by wire = # turns + 1 more**
 - ♦ 100 turns of wire actually occupies 101 turns of area.
- **On Toroids:**
 - **50% full is difficult. 75% is a practical maximum.**
 - ♦ Turns for one layer on ID is lower than one layer on OD
 - ♦ The effective ID for toroid is minimum ID – one wire diameter – Bulge factor
- **Wire**
 - **The bulge factor for wire usually $\frac{1}{2}$ the wire diameter + 10 mil. Bulge factors are not added for each layer of wire. It is only added on the first layer of wire and when the wire diameter increases on a later winding.**
 - ♦ Example: For a 30 mil diameter wire, the bulge factor is 25 mils (30/2 + 10). When a 40 mil diameter wire is wound on top of the 30 mil wire, the “running” bulge factor increases from 25 mil to 30 mil (40/2+10).

Summary of Equations

$$\frac{(LI)^2}{DCR} = Kc * (Ur H)^2 * \frac{Ac^2 Aw}{MTL} * \frac{1}{pd^2} * \frac{Ap}{Aw}$$

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$$\frac{Ac^2 Aw}{MTL} = \frac{pd^2 * (LI)^2 / DCR}{Kc * (Ur H)^2 * Ap / Aw}$$

$$\frac{(LI)^2}{DCR} = Kc * (Ur H)^2 * \frac{Ac^2 Aw}{MTL} * \frac{1}{pd^2} * \frac{Ap}{Aw}$$

$$N = Kn * (LI) / ((Ur H) (Ac))$$

$$Ur = Ku * L * lm / (Ac N^2)$$

$$la = F * lm * (Um - Ur) / (Um Ur)$$

$$F = 1 + (2 * lg / Kstk) * Ac^{.5} * \ln(2 * G / lg)$$

Wire Diameter:

$$\text{Diameter} = \text{square root} [AW * (Ap / Aw) / (N * \text{strands})]$$

$$\text{Dia (mils)} = \text{square root} [(PI / 4) * (\text{Cir mils}) * (Ap / Aw) / (N * \text{strands})]$$

Units for equations

Kc	Kn	Ku	Ac	Aw	MTL	lm	p	d ²
49.95E-12	15.5E6	31.33E6	in ²	in ²	in	in	mOhm/ft	in ²
1.200E-12	100.E6	79.58E6	cm ²	in ²	in	cm	mOhm/ft	in ²
942.5E-21	100.E6	79.58E6	cm ²	cir mil	in	cm	mOhm/ft	in ²
472.4E-15	100.E6	79.58E6	cm ²	cm ²	cm	cm	mOhm/ft	in ²
100.0E-18	100.E6	79.58E6	cm ²	cm ²	cm	cm	Ohm/cm	cm ²

Note: If p or d is in cm, a new table of pd² must be generated.

From "Single Iteration Magnetic Design Based On Winding Resistance"

Kstk is magnetic area of the core / physical area of core. For Ferrite and Powered iron, this is 1.

Summary of Equations

$$\frac{(LI)^2}{DCR} = Kc * (Ur H)^2 * \frac{Ac^2 Aw}{MTL} * \frac{1}{pd^2} * \frac{Ap}{Aw}$$

UrH in gauss for various materials

Material	80% Ur	50% Ur	Notes
14-300u	2100	3000	MPP cores - Ur stable with gauss
550u	750	1400	550u good for high inductance designs
Steel	10000	15000	Use 80% left gauss for high Ur or greater than 25C operation. Use 50% left gauss for low Ur or 25C operation.
Alloy 49	9000	12000	
Ferrite	1800	2700	

pd² for various wire types (p in mOhm/ft, d in inches)

AWG #	pd ² Round Solid Bare Copper	pd ² Round Solid Double Film	pd ² Square Solid Double Film	pd ² Round Solid 81822 Teflon	pd ² Round Stranded Teflon	pd ² Round Aluminum Double Film
4	.01037	.01094	.00901			.01770
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36	.01037	.01646				
40	.01037	.01726				

Note: pd² for copper foil without insulation = .00814

From "Single Iteration Magnetic Design Based On Winding Resistance"

Note: Normally, the BH curve isn't used for MPPs. You use Ur under bias from the DC bias curve for the MPP to get UrH (gauss).

Summary of Equations

$$\frac{(LI)^2}{DCR} = Kc * (Ur H)^2 * \frac{Ac^2 Aw}{MTL} * \frac{1}{pd^2} * \frac{Ap}{Aw}$$

MPP TOROID CORES

OD MAX INCH	ID MIN INCH	HT MAX INCH	LM CM	AC ₂ CM ²	AW INCH ²	50% MTL INCH	AL Ur	IN ⁵ SINGLE STACK	IN ⁵ DOUBLE STACK
0.150	0.060	0.072	0.817	0.0137	0.0028	0.272	0.207	46.9E-09	123.E-09
0.165	0.078	0.110	0.942	0.0211	0.0048	0.357	0.280	143.E-09	354.E-09
0.205	0.076	0.130	1.060	0.0285	0.0045	0.437	0.330	203.E-09	508.E-09
0.275	0.090	0.135	1.361	0.0470	0.0064	0.511	0.400	661.E-09	1.73E-06
0.285	0.090	0.125	1.363	0.0476	0.0064	0.500	0.433	692.E-09	1.85E-06
0.288	0.087	0.218	1.363	0.0920	0.0059	0.690	0.823	1.75E-06	4.29E-06
0.335	0.135	0.150	1.787	0.0615	0.0143	0.585	0.413	2.22E-06	5.88E-06
0.405	0.168	0.150	2.180	0.0752	0.0222	0.644	0.427	4.68E-06	12.8E-06
0.405	0.168	0.180	2.180	0.0945	0.0222	0.704	0.530	6.76E-06	17.9E-06
0.425	0.180	0.180	2.380	0.1000	0.0254	0.720	0.530	8.50E-06	22.7E-06
0.468	0.232	0.186	2.690	0.0906	0.0423	0.759	0.423	11.0E-06	29.5E-06
0.530	0.275	0.217	3.120	0.1140	0.0594	0.869	0.447	21.3E-06	56.9E-06
0.680	0.375	0.280	4.110	0.1920	0.1104	1.112	0.577	87.9E-06	234.E-06
0.830	0.475	0.280	5.090	0.2260	0.1772	1.230	0.543	177.E-06	486.E-06
0.930	0.527	0.330	5.670	0.3310	0.2181	1.412	0.720	407.E-06	1.11E-03
0.956	0.542	0.382	5.880	0.3880	0.2307	1.537	0.563	543.E-06	1.45E-03
1.090	0.555	0.472	6.350	0.6540	0.2419	1.841	1.257	1.35E-03	3.57E-03
1.385	0.888	0.387	8.950	0.4540	0.6193	1.871	0.633	1.64E-03	4.64E-03
1.332	0.760	0.457	8.150	0.6720	0.4536	1.990	1.017	2.47E-03	6.78E-03
1.445	0.848	0.444	8.980	0.6780	0.5648	2.050	0.937	3.04E-03	8.49E-03
1.602	0.918	0.605	9.840	1.0720	0.6619	2.503	1.343	7.30E-03	19.7E-03
1.875	1.098	0.635	11.630	1.3400	0.9469	2.778	1.423	14.7E-03	40.4E-03
2.035	1.218	0.565	12.730	1.2500	1.1652	2.761	1.217	15.8E-03	45.0E-03
1.875	0.918	0.745	10.740	1.9900	0.6619	3.043	2.247	20.7E-03	55.6E-03
2.285	1.368	0.585	14.300	1.4440	1.4698	3.001	1.247	24.5E-03	70.6E-03
3.108	1.888	0.550	19.600	1.7700	2.7996	3.584	1.137	58.8E-03	180.E-03

Design Example 1

$$\frac{(LI)^2}{DCR} = K_c * (U_r H)^2 * \frac{A_c^2 A_w}{M_T L} * \frac{1}{p d^2} * \frac{A_p}{A_w}$$

DCR < .020 Ohms, L = 25E-6 Henry at 6.6 Amps. The DC resistance and size is critical. The inductance is desired to swing from some higher value to approximately 25E-6 Henries. A MPP is desired, so set UrH = 2800 gauss for a 50% swing in inductance.

A.
$$\frac{A_c^2 A_w}{M_T L} = \frac{p d^2 * (LI)^2 / DCR}{K_c * (U_r * H)^2 * A_p / A_w} = \frac{.013 * (25E-6 * 6.6)^2 / .02}{50E-12 * 2800^2 * .4} = 1.1E-4$$

0.68 OD toroid's in⁵ = 8.97E-5 0.83 OD toroid's in⁵ = 1.81E-4

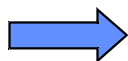
The 0.68 OD toroid is chosen because of size considerations.

0.680 OD, 0.375 ID, 0.280 HT

Ac = .0298 sq in, Aw = .11 sq in, lm = 1.618 in, MTL = 1.09 in

MPP TOROID CORES

OD MAX INCH	ID MIN INCH	HT MAX INCH	LM CM	AC ₂ CM ²	AW INCH ²	50% MTL INCH	AL Ur	IN ⁵ SINGLE STACK	IN ⁵ DOUBLE STACK
0.405	0.168	0.180	2.180	0.0945	0.0222	0.704	0.530	6.76E-06	17.9E-06
0.425	0.180	0.180	2.380	0.1000	0.0254	0.720	0.530	8.50E-06	22.7E-06
0.468	0.232	0.186	2.690	0.0906	0.0423	0.759	0.423	11.0E-06	29.5E-06
0.530	0.275	0.217	3.120	0.1140	0.0594	0.869	0.447	21.3E-06	56.9E-06
0.680	0.375	0.280	4.110	0.1920	0.1104	1.112	0.577	87.9E-06	234.E-06
0.830	0.475	0.280	5.090	0.2260	0.1772	1.230	0.543	177.E-06	486.E-06
0.930	0.527	0.330	5.670	0.3310	0.2181	1.412	0.720	407.E-06	1.11E-03
0.956	0.542	0.382	5.880	0.3880	0.2307	1.537	0.563	543.E-06	1.45E-03



From "Single Iteration Magnetic Design Based On Winding Resistance"

Design Example 1

$$\frac{(LI)^2}{DCR} = K_c * (U_r H)^2 * \frac{A_c^2 A_w}{MTL} * \frac{1}{pd^2} * \frac{A_p}{A_w}$$

B. Determine N and U_r

$$N = K_n * (LI) / ((U_r H) (A_c)) = 15.5E6 * 25E-6 * 6.6 / (2800 * .0298) = 30 \text{ turns}$$

$$U_r = K_u * \frac{L_{lm}}{A_c N^2} = 31.3E6 * \frac{25E-6 * 1.61}{.030 * 30^2} = 46 \text{ u at bias}$$

Because the design is for 50% left, actual permeability of the core should be:

$$46 / (50\% / 100\%) = 92 \text{ u the nearest perm is 125u.}$$

$$\text{Wire diameter} = (0.4 * 0.11 / 30)^{.5} = .038 \text{ diameter} = \#19 \text{ AWG}$$

To allow for winding variances, use a wire diameter halfway between the actual diameter and the next larger size.

$$\#19 \text{ winding diameter} = (.043 + .039) / 2 = .0414 \text{ inches}$$

$$\text{Maximum turns for 1 layer in ID} = \text{PI} * (.375 - .0414) / .0414 = 25.3$$

Round down to 24 turns for ease of manufacturing/lower cost.

Design Example 1

$$\frac{(LI)^2}{DCR} = K_c * (U_r H)^2 * \frac{A_c^2 A_w}{MTL} * \frac{1}{pd^2} * \frac{A_p}{A_w}$$

C. Calculate parameters using 24 turns #19 on Mag. Inc. 55120-A2 core.

$$24 * 1.09"/\text{turn} * 1\text{ft}/12" * 8.046 \text{ Ohms}/1000 \text{ ft} = .0175 \text{ Ohms Nom}$$

$$L \text{ at zero DC} = 72\text{mH} * (24/1000)^2 = 41.5 \text{ uH}$$

The core tolerance is $\pm 8\%$.

Allow $\pm 10\%$ at zero DC for test and production variance.

The inductance will be lower than desired, so the ripple current will be higher—test the inductance with DC at 7.0 Amps.

$$O_e = .4 \text{ PI} * 24 * 7/4.11 = 51.4 \text{ oersteds} = 46\% \text{ left}$$

$$L \text{ min at } 7\text{A} = 41.5 \text{ uH} * 90\% * 46\% = 17 \text{ uH min}$$

D. Test Results:

$$L \text{ at } 10\text{kHz zero DC} = 43.5 \text{ uH} \quad Q = 84$$

$$L \text{ at } 10\text{kHz } 7.0\text{A DC} = 22.8 \text{ uH}$$

$$DCR = .016 \text{ Ohms with } 1.5 \text{ inch leads}$$

$$51.4 \text{ oe} = 40.88 \text{ NI/cm}$$

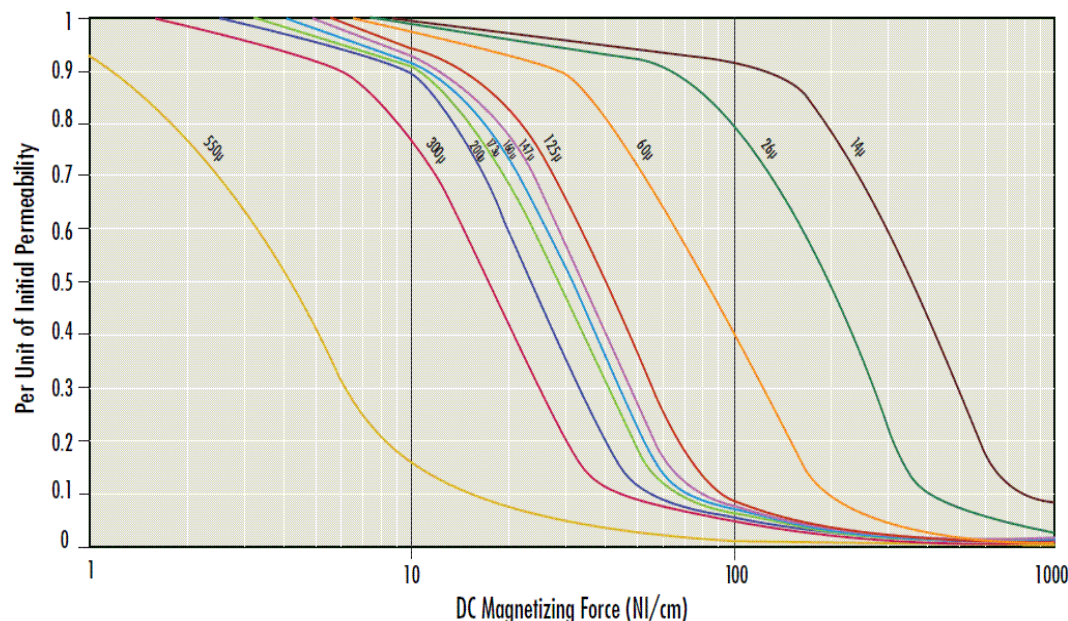
Often the “swing” on the MPP is margined by 10%.

- If the curve says the part drops by 10%, we use 11% on the spec.

- If the part drops by 50%, we use 55% on the spec.

- 1 more turn would be $\sim 17\text{uH} * (25\text{T}/24\text{T})^2 = 18.4 \text{ uH}$ before the additional bias drop is added in. This indicates 24T is a good choice.

MPP



Design Example 2

$$\frac{(LI)^2}{DCR} = K_c * (U_r H)^2 * \frac{A_c^2 A_w}{MTL} * \frac{1}{pd^2} * \frac{A_p}{A_w}$$

Example 2: Design a coupled inductor to meet a given size

The customer needs a coupled inductor that must fit on an E375 E-core. The four outputs are 3.9 Amps at +5V, 3.9 Amps at +5V, 3.1 Amps at -5V and 3.1 Amps at -5V. The transformer that drives the inductor has a .125 Volt IR drop. Therefore, assume that a .125 Volt drop can be tolerated in the inductor. The operating frequency is 80 kHz. The customer wants the maximum inductance he can get out of that size.

A. Reflect all currents into one winding. When doing this assume 0.8 volt diode drop in series with each winding.

$$\begin{aligned} \text{VA effective} &= (5V + .8V) * 3.9A &= 22.62 \text{ VA} \\ &+ (5V + .8V) * 3.9A &= 22.62 \text{ VA} \\ &+ (5V + .8V) * 3.1A &= 17.98 \text{ VA} \\ &+ (5V + .8V) * 3.1A &= \underline{17.98 \text{ VA}} \\ \text{VA effective} &= &81.20 \text{ VA} \end{aligned}$$

$$\text{Effective DC current} = 81.20 \text{ VA} / (5V + .8V) = 14.0 \text{ Amps DC}$$

Assume an AC ripple current of 10% of the DC (zero to peak).

$$\text{Peak AC current} = 1.4 \text{ Amps.} \quad \text{RMS AC current} = 0.81 \text{ Amps}$$

$$\text{Design current} = 14.0 + 1.40 = 15.4 \text{ Amps (peak)}$$

B. Calculate DCR limit: $.125V / 14 \text{ Amps} = .0089 \text{ Ohm}$

Design Example 2

$$\frac{(LI)^2}{DCR} = K_c * (U_r H)^2 * \frac{A_c^2 A_w}{MTL} * \frac{1}{pd^2} * \frac{A_p}{A_w}$$

C. Rearrange the $(LI)^2/DCR$ equation to solve for L

$$(15) \frac{(LI)^2}{DCR} = K_c * (U_r * H)^2 * \frac{A_c^2 A_w}{MTL} * \frac{1}{pd^2} * \frac{A_p}{A_w}$$

$$L^2 = (DCR/I^2) * K_c * (U_r H)^2 * A_c^2 A_w / MTL * 1/pd^2 * A_p/A_w$$

$$L^2 = .0089/15.4^2 * 50E-12 * 3000^2 * A_c^2 A_w / MTL * 1/.013 * .8$$

$$\implies L^2 = 1.039E-6 * A_c^2 A_w / MTL$$

For the E375 core with a PC mount bobbin:

$$A_c = .135 \text{ in}^2, A_w = .147 \text{ in}^2, MTL = 2.89", l_m = 2.717"$$

$$A_c^2 A_w / MTL = .135^2 * .147 / 2.89 = 927E-6 \text{ in}^5$$

$$L^2 = 1.039E-6 * 927E-6 = 962E-12 \implies L = 31E-6 \text{ Henry}$$

The actual zero to peak current will be:

$$I = (5.8V / (80Khz * 31E-6 \text{ Henry})) / 2 = 1.17A$$

Which is less than the 1.4 Amps estimated.

Design Example 2

$$\frac{(LI)^2}{DCR} = K_c * (U_r H)^2 * \frac{Ac^2 Aw}{MTL} * \frac{1}{pd^2} * \frac{Ap}{Aw}$$

D. Solve for N, Ur, la, and F

$$(18) N = K_n * (LI) / ((U_r H) (Ac))$$

$$N = 15.5E6 * 31E-6 * 15.4 / (3000 * .135) = 18.39 = 18$$

$$(20) U_r = K_u * \frac{L l_m}{Ac N^2}$$

$$U_r = 31.33E6 * 31E-6 * 2.717 / (18^2 * .135) = 60.33$$

$$(21) l_a = F * l_m * (U_m - U_r) / (U_m U_r)$$

$$l_a = 1 * 2.717 * (1000 - 60) / (1000 * 60) = .042 \text{ inch total} \\ = .021 \text{ per leg}$$

$$(23) F = L' / L = 1 + \frac{1}{K_{stk}} * \frac{2 * l_g}{Ac \cdot 5} * \ln \frac{2 G}{l_g}$$

$$F = 1 + 1 * (2 * .021 / (.135 \cdot 5)) * \ln(2 * .76 / .021) = 1.49$$

Final air gap in each leg = 1.49 * .021 = .031 inch in each leg.

From "Single Iteration Magnetic Design Based On Winding Resistance"

Design Example 2

$$\frac{(LI)^2}{DCR} = Kc * (Ur H)^2 * \frac{Ac^2 Aw}{MTL} * \frac{1}{pd^2} * \frac{Ap}{Aw}$$

E. Calculate actual DCR

$$\text{Dia wire} = (.147 * .8 / (18 * 4 \text{ strands}))^{.5} = .0404 \text{ inches}$$

The measured bobbin winding window is .650 inch long by .260 inch tall.

$$.65 / .04 = 16 \text{ wires}$$

$$.26 / .04 = 6.5 \text{ wires}$$

Only 16 wires can fit in one layer of the bobbin. Since the number of wires in a layer wind equals 1 + the number of turns, only 15 turns of .0404 inch wire will fit in one layer. Partial layers (1.01 to 1.6 layers) are not desirable.

$$.65 / 19 \text{ wires} = .0342 \text{ diameter wire or \#21}$$

$$(.260 - .020 \text{ inches tape}) / .034 = 7 \text{ layers of wire}$$

A multiple of 4 layers with 2.5 mils tape between layers is desired.

$$(.260 - .020 \text{ inches tape}) / 8 \text{ layers} = .030 \text{ diameter wire—\#22}$$

$0.65 / .029 = 22$ wires or a total of 21 turns can be wound. Eighteen turns are wound for lower DCR. Note: the customer requested a sample of both 18 and 21 turns. Only data for 18 turns is presented.

$$18 \text{ turns} * 2.89 \text{ in/turn} * 1 \text{ft} / 12 \text{ in} * .0162 \text{ Ohm/ft} = .070 \text{ Ohms}$$

Equivalent total wire resistance will be $.070 \text{ Ohms} / 8 = .0088 \text{ Ohms}$.

Since the inner layers of wire will have less resistance than the outer layers, use the inner layers for the higher current windings.

The wire is wound in multiple single layers to minimize the capacitance across the windings for low coupled noise and low capacitive drive losses in the switching components.

From "Single Iteration Magnetic Design Based On Winding Resistance"

Design Example 2

$$\frac{(LI)^2}{DCR} = K_c * (U_r H)^2 * \frac{A_c^2 A_w}{MTL} * \frac{1}{pd^2} * \frac{A_p}{A_w}$$

F. Check temperature rise

The bobbin is 0.990 inch high, 0.990 inch wide, and 0.732 inch long.
The surface area is:

$$4 * .99 * .732 + 2 * .990^2 = 4.86 \text{ square inches}$$

$$\begin{aligned} \text{Resistive loss} &= .81^2 \text{ A} * .0088 \text{ Ohms} = 0.01 \text{ Watts} \\ &+ 3.90^2 \text{ A} * .070/2 \text{ Ohms} = .53 \text{ Watts} \\ &+ 3.90^2 \text{ A} * .070/2 \text{ Ohms} = .53 \text{ Watts} \\ &+ 3.10^2 \text{ A} * .070/2 \text{ Ohms} = .34 \text{ Watts} \\ &+ 3.10^2 \text{ A} * .070/2 \text{ Ohms} = \underline{.34 \text{ Watts}} \\ \text{Total IR loss} &= 1.74 \text{ Watts} \end{aligned}$$

$$\begin{aligned} \text{AC gauss} &= 5.8E8 / (4.00 * 18 \text{ turns} * 80\text{kHz} * .904 \text{ cm}^2) \\ &= 111 \text{ gauss} \end{aligned}$$

$$\text{Core loss} = 6.23 \text{ cm}^3 * .001 \text{ Watt/cm}^3 = .006 \text{ Watts}$$

$$\text{Temperature rise} = 80 * (4.86 \text{ in}^2)^{-.7} * (1.75\text{W})^{+.85} = 42 \text{ C rise}$$

High, but OK.

The 42 C rise is on the surface of the core; the windings will actually be warmer. This temperature is acceptable because heat will also flow out the leads to the PC board. When the coil is potted, the core will also draw some heat away.

Design Example 2

$$\frac{(LI)^2}{DCR} = K_c * (U_r H)^2 * \frac{A_c^2 A_w}{M L} * \frac{1}{p d^2} * \frac{A_p}{A_w}$$

G. Test Results

The inductance for a .031 inch gap in each leg is given below for the core at 25C and 125C.

DC current	5	7	9	10	15	16	17	18	19	20	Amps
25 C Ind	29.6	30.5	32.7	33.2	33.6	33.2	32.7	31.6	26.0	16.8	uH
125 C Ind	-	-	-	-	30.7	25.1	-	-	-	7.5	uH
L at 1V 75 Khz = 35.3 uH, Q = 140, Rp = 2.3 KOhm											
L at 1V 100 Khz = 35.3 uH, Q = 150, Rp = 3.3 KOhm											

Below are the results of a square stack of 6 mil Alloy 49 (50% Nickel) with a .010 inch gap in each leg on the same bobbin.

DC current	5	15	16	17	20	Amps	
25 C Ind	110	86	53	32	-	uH	.008 gap
25 C Ind	94.8	-	85.6	-	61	uH	.010 gap
125 C Ind	98.4	-	82.8	-	41	uH	.010 gap
L at 1V 75 Khz = 85.6 uH, Q = 3.45, Rp = 148 Ohm, .010 gap							
L at 1V 100 Khz = 82.0 uH, Q = 3.07, Rp = 166 Ohm, .010 gap							
L at 20 hz = 147 uH L at 150 Khz = 76.4 uH							

Note: Rp changing from 2.3K to 148 ohm means core loss is 15 times higher on Alloy 49. The design can accept the higher core loss. Core loss going from 6 mW to 6 mW * 150/3 = 300 mW is OK.

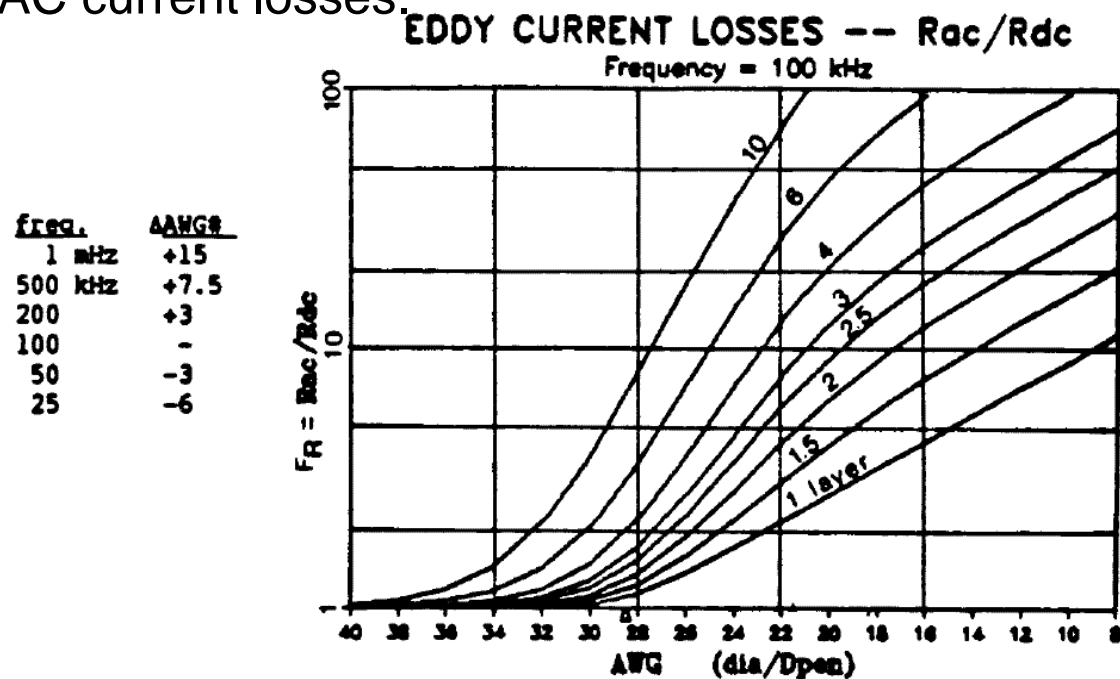
From "Single Iteration Magnetic Design Based On Winding Resistance"

AC Resistance / Eddy Current

$$\frac{(LI)^2}{DCR} = Kc * (Ur H)^2 * \frac{Ac^2 Aw}{MTL} * \frac{1}{pd^2} * \frac{Ap}{Aw}$$

At high frequencies, only the surface of the wire is used to conduct current. As the number of layers increases, this effect becomes more pronounced. To counter this effect, litz wire is used and/or winding interleaving tricks. Litz wire and winding interleaving tricks will result in a smaller fill factor for the magnetic which can lead to needing a larger core.

The Eddy Current Effects inside transformer windings was originally covered by an article written by P.L. Dowell in 1966 in the proceedings of the IRE. The Unitrode app note SLUP197 (See curve below) has a very good explanation of how to analyze these AC current losses.

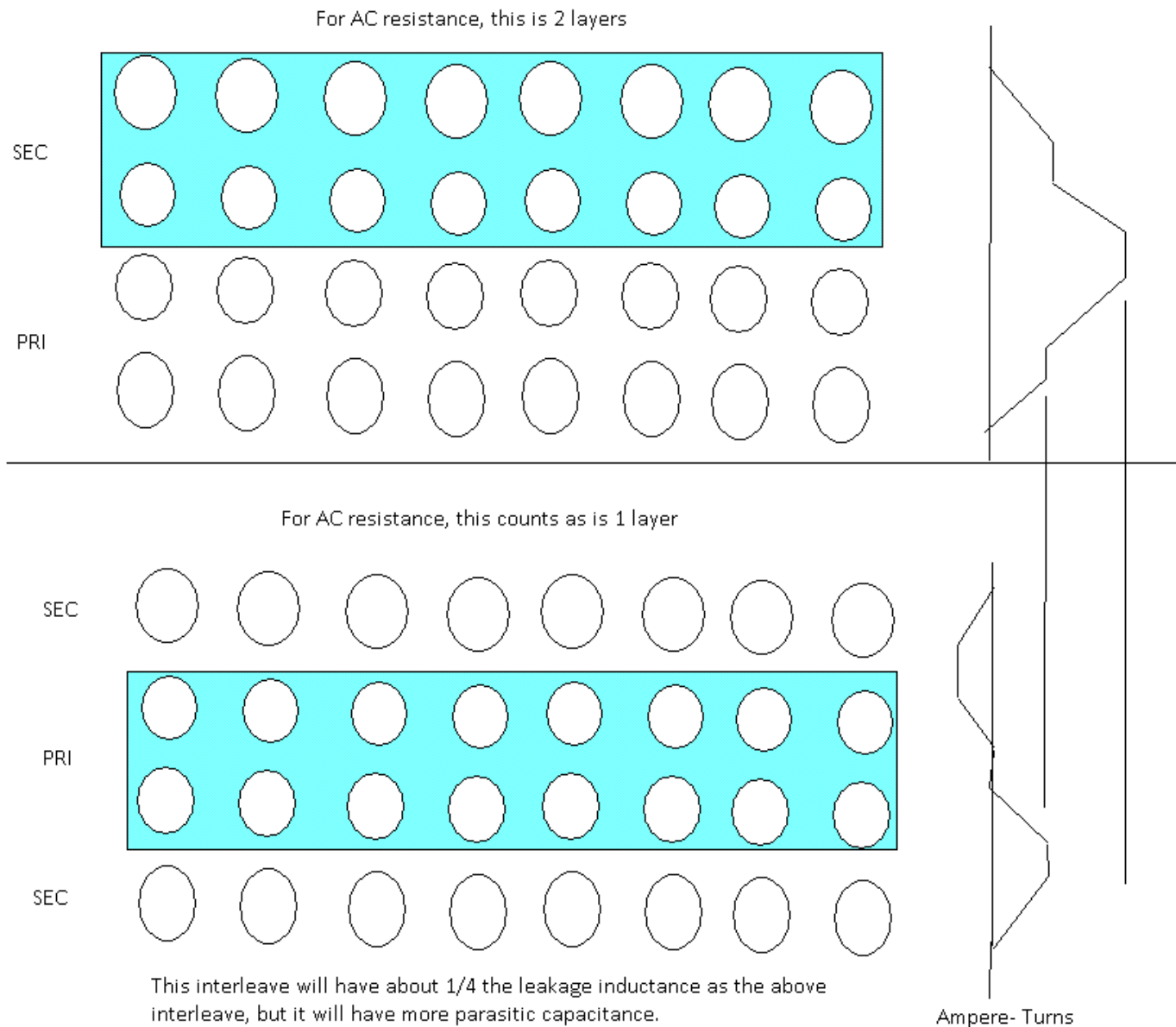


AC Resistance / Eddy Current

$$\frac{(LI)^2}{DCR} = K_c * (U_r H)^2 * \frac{A_c^2 A_w}{MTL} * \frac{1}{pd^2} * \frac{A_p}{A_w}$$

Cross section of a winding

Interleaving the windings can reduce both the AC resistance (Lower effective layer) and reduce the leakage inductance at the expense of added capacitance.



TANSTAAFFV: There Ain't No Such Thing As A Free Variable (but many are cheap enough to not worry about.)

See the P.L. Dowell article and Unitrode Articles on Eddy Currents for full explanation.

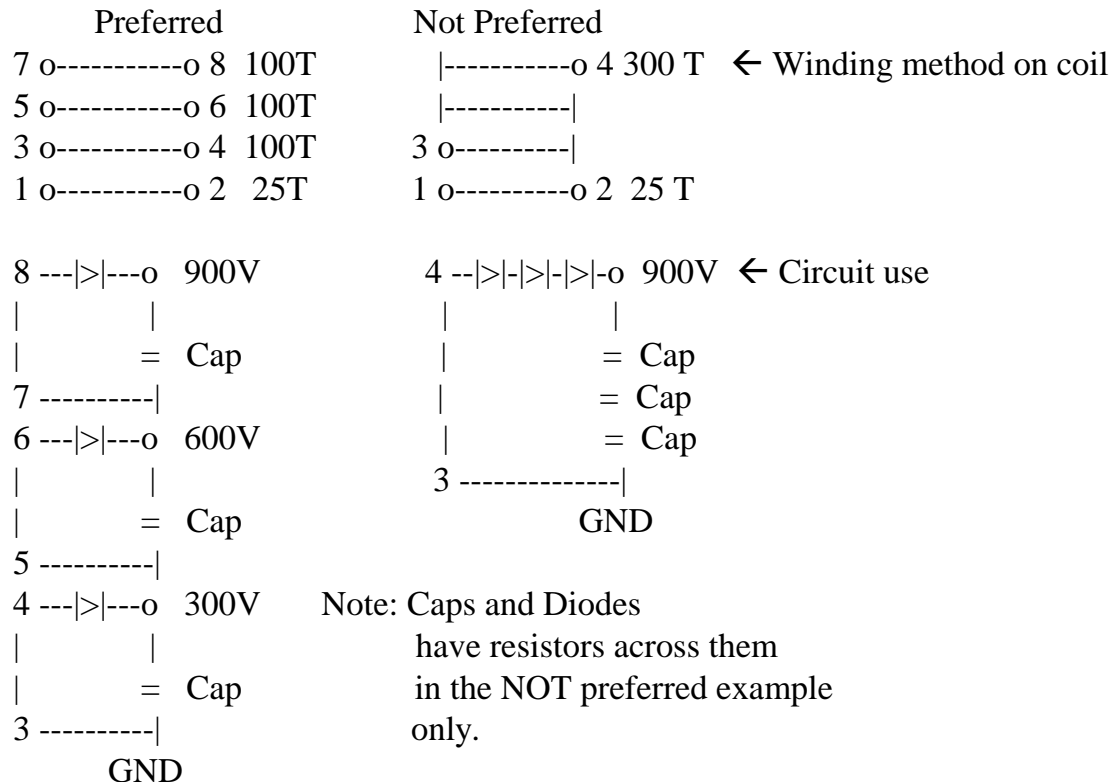
Insights – Winding Techniques

$$\frac{(LI)^2}{DCR} = K_c * (U_r H)^2 * \frac{A_c^2 A_w}{MTL} * \frac{1}{pd^2} * \frac{A_p}{A_w}$$

- **Budget for winding techniques with Ap/Aw**

- A taste of how important winding techniques are. Consider a high voltage flyback. Reducing energy stored in the capacitance is critical or all the energy ($0.5 L I^2$) in the primary ends up in driving parasitic capacitance ($0.5 C V^2$).

- First add insulation between the winding and the core to reduce capacitance.
- Then reduce the delta V across capacitance as much as possible. 900V across 300pF is 121.5uJ of stored energy. Dividing the winding into three 300V changes across 100pF is 13.5uJ of stored energy. ($0.5 * 900V^2 * 300pF$ vs. $3X * 0.5 * 300V^2 * 100pF$)
- Other winding techniques can drop the effective parasitic capacitance further.



Magnetic Production Issues

$$\frac{(LI)^2}{DCR} = K_c * (U_r H)^2 * \frac{A_c^2 A_w}{MTL} * \frac{1}{pd^2} * \frac{A_p}{A_w}$$

- **Taping a toroid with 3M #5 tape (acrylic adhesive)**
 - Reduces winding capacitance to core
 - Reduces part cracking in temperature cycling and handling
 - **Over all, acrylic adhesive works the best.**
 - ◆ Silicon adhesive doesn't cushion, but is easiest to work with in the magnetic factory.
 - ◆ Silicon adhesive contaminates solder washes and can prevent good solder joints. I.E. the PWB assembly houses don't want you to use it.
 - ◆ Rubber adhesive doesn't cushion and seems to make cracking worse. But has good initial adhesion
 - ◆ Rubber adhesive isn't always hydrolytically stable. When exposed to high humidity for long durations and some cleaning solvents, it turns to goo and no longer sticks
- **Don't let thick wires touch thin wires.**
 - **6 AWG is limit for directly touching. 10 AWG is limit with precautions such as taping between contacts. > 10 AWG requires extreme precautions (taping, cushioning etc.)**
 - ◆ The thicker wire can crush or stretch thinner wire both on coil and on lead outs
 - ◆ Thinner wire will have excessive neck down during soldering.
 - ◆ Generally, each lead out of the magnetic must support full weight of part to prevent damage in shipping, handling and installation.

Inductor Considerations

$$\frac{(LI)^2}{DCR} = K_c * (U_r H)^2 * \frac{A_c^2 A_w}{MTL} * \frac{1}{pd^2} * \frac{A_p}{A_w}$$

- Temperature rise is dependent not only on the resistive loss (with eddy current effects) and the core loss, but also on the method of getting the heat out of the device. So lets not use temperature rise in the core selection method.
- Insulation breakdown and self resonant frequency are dependent on the insulation thickness, dielectric constant, surface area of the winding, fill factor and winding method. This is addressed with A_p/A_w .
- Q is dependent on total AC losses, frequency and gauss drive (usually lower gauss drive gives a higher Q).

$$1/Q \text{ total} = 1/Q \text{ series [DCR]} + 1/Q \text{ parallel [core]} + 1/Q \text{ load}$$

$$Q \text{ series} = 2\pi * F * L / DCR$$

$$Q \text{ Parallel} = R_{\text{core loss}} / (2\pi * F * L)$$

$$Q \text{ load} = R_{\text{load}} / (2\pi * F * L)$$

Inductor Size Drivers

$$\frac{(LI)^2}{DCR} = K_c * (U_r H)^2 * \frac{A_c^2 A_w}{MTL} * \frac{1}{pd^2} * \frac{A_p}{A_w}$$

For a smaller part allow:

- Larger drops in inductance with applied current (i.e. higher Gauss)
- Higher DC resistance of the windings
- Higher allowable temperature rise
- Lower thermal impedance to the final heat sink